CIVIL ENGINEERING

NAGAR PARISHAD

TECHNICAL SUBJECTS

- स्थापत्य अभियांत्रिकी सेवा (गट–क)
  (श्रेणी अ,ब आणि क)

संपूर्ण मार्गदर्शक

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- For better understanding arranged in simple & Easy language
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CIVIL ENGINEERING

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- सर्व टेक्निकल विषयांचा समावेश
- सर्व विषयांचे वस्तुनिष्ठ बहुपर्यायी प्रश्न
- १००० पेक्षा जास्त प्रश्नाचा समावेश
- उत्तम आकलनासाठी साध्या व सोप्या भाषेत मांडणी
- जलद उजळणी साठी उपयुक्त

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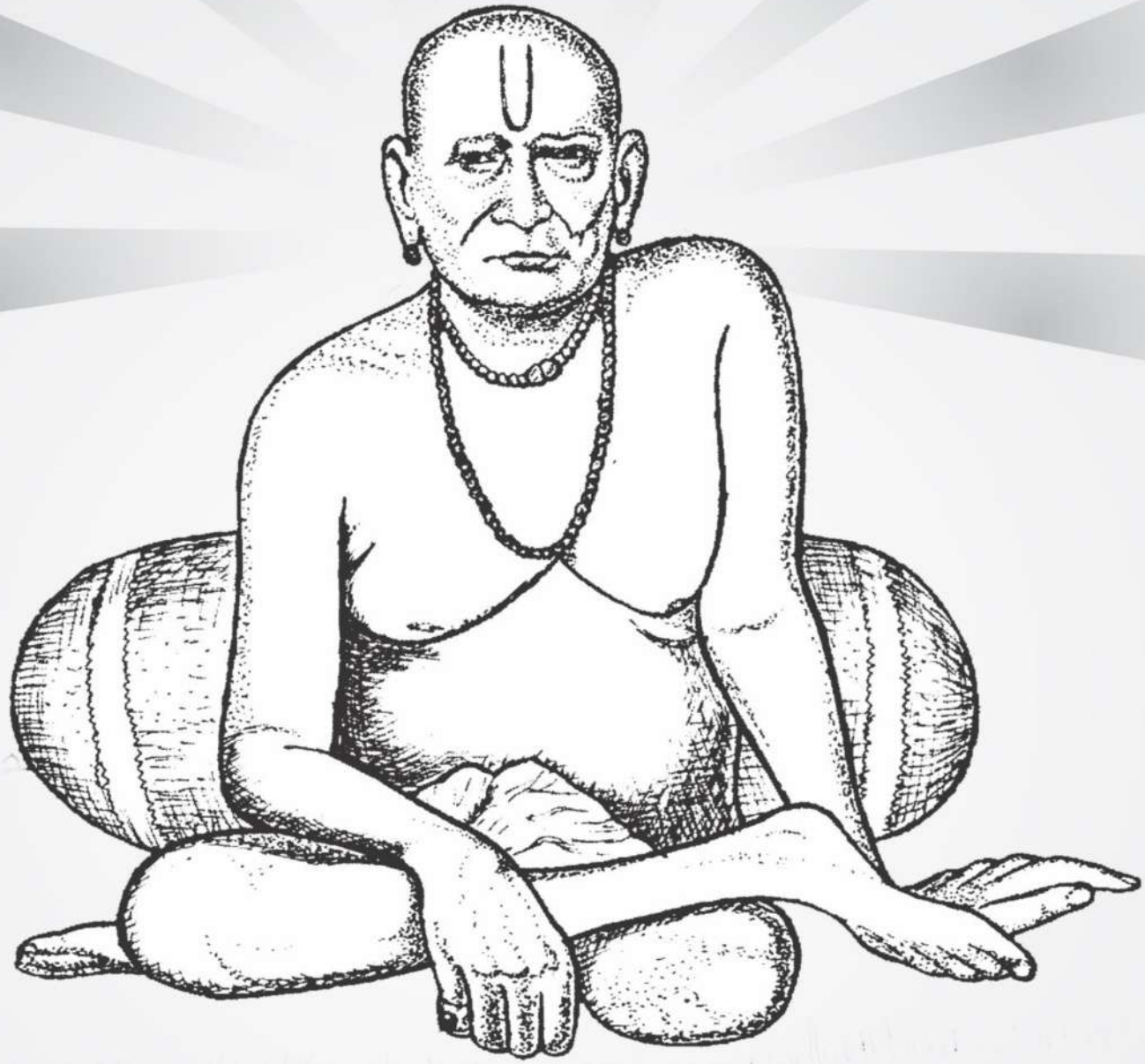

श्री स्वामी समर्थ...

स्वामींच्या चरणी अर्पण ...

INDEX

CIVIL ENGINEERING (TECHNICAL)

BUILDING MATERIALS & CONSTRUCTION

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1.CEMENT

Introduction

A mixture of silicates and aluminates of calcium that when mixed with water it binds a stone-sand mixture into a strong concrete within a few days.

Mixing of Raw Materials

The raw materials used in the manufacture of cement are

(a) Argillaceous material consisting of silicates of alumina in the form of clays and shales.

(b) Calcareous materials - in the form of limestone, chalk and marl, which is mixture of clay and calcium carbonate. These materials are mixed thoroughly.

Burning

Burning is carried out in a rotary kiln.

The material is then heated to a temperature of 1400°C to 1500°C and the formation of clinckers take place.

Grinding

. During grinding, a small quantity of gypsum about (3% to 4%) is added.

Gypsum controls the initial setting time of cement.

Gypsum acts as a retarder, and delays the initial setting action of cement.

Chemical Composition: & it's Function of Ingredients of Cement.

Sr	Oxide	Proportion
1	Lime-(C_aO)	60-65%
2	Silica-(S_iO_2)	16-25%
3	Alumina-(Al_2O_3)	3-8%
4	Ferrous (Fe_2O_3)	0.5-6%
5	Magnesia (MgO)	0.1-4%

1) Lime-(C_aO)

Chemical composition-$60-65\%$

Lime is primarily cementing property in cement.

Function:-

i) It makes the cement sound and strong.

ii) It controls strength and soundness.

iii) The lime is excess quantity makes the cement unsound and causes the cement to expand and disintegrate

2) Silica-(S_iO_2)

Chemical composition-$16-25\%$

Function:-

i) Gives strength to the cement due to the formation of dicalcium and tricalcium silicates.

ii) Silica in excess quantity causes slow setting of cement.

3) Alumina-(Al_2O_3)

Composition-$3-8\%$

It is major constituent in Argillaceous materials.

Function:-

i) It provides quick setting property.

ii) Lowers the clinkering temperatures.

iii) Alumina excess lowers strength.

4) Iron Oxide-(Fe_2O_3)

Composition-$0.5-6\%$

Function:-

i) Provide colour, hardness and strength to cement.

ii) Also help in fusion of raw material during the manufacturing of cement at high temperature.

iii) Impart raddish brown tint in cement.

5) Magnesium Oxide-(M_gO)

Composition-$0.1-4\%$

Function:-

i) It provides hardness and colour to the cement when present in small quantity.

ii) It excess cause the cement unsound & cracking.

6) Alkalies-(Traces) (Soda & Potash)

These should be present in small quantities.

Excess quantities will causes efflorescence and cracking & staining of structure means-development of white grey spots over the surface of structure.

Composition of cement clinker

The cement clinkers (which are formed when calcareous & argillaceous raw materials are mixed and burned in rotary kilns).

SR.		Bogue's Compound	Composition
1	Tricalcium silicate (C_3S)	Alite	40-45%
2	Dicalcium silicate (C_2S)	Belite	25%
3	Tricalcium Aluminate (C_3A)	Celite	11%
4	Tetracalcium Alumino ferrite (C_4AF)	Felite	9%

1) Tricalcium Silicate (C_3S)- Alite-45%

It develops high early strength at first 7 days.
The presence of C_3S in cement hydrates more rapidly.
It generates more heat of hydration (120 cal/gm).
It posses less resistance to sulphate attack.

2) Dicalcium Silicate (C_2S)- Belite-25%

It develops later strength after 7 days.
Cement hydrates slowly.
Less heat of hydration (60 Cal/gm.).
It provides good ultimates strength to cement.
It offers more resistance to sulphate attack.

3) Tricalcium Aluminate (C_3A)- Clite-11%

It causes initial setting of cement.
It react fast with water & generates large amount of heat of hydration (320cal/gm).
It is the first compound with reacts with water when mixed with cement.
C_3A Bogue's compound responsible for flash set.

4) Tetra-Calcium Alumino Ferrite (C_4AF)-Felite-9%

It has poor cementing value & cause initial setting of cement.
It reacts slowly with water & generates small amount of heat hydration.

Hydration of cement

The heat of hydration is determined by adiabatic calorimeter test or vacuum flask test.

The heat of hydration f0or OPC at $7\ days < 65cal/gm$ and at $28\ days < 75cal/gm$

Types of cement:

1) Ordinary Portland Cement-$(IS\ 269-1976)$:

Used for general construction.
Portland Cement contain

Sr	Content	Proportion
1	C_3S	50%
2	C_2S	25%
3	C_3A	11%
4	C_4AF	9%

In OPC, Initial setting time-30minutes & Final setting time-10hrs.

Compressive strength of OPC

	Grade 33	Grade 43	Grade 53
3 Days	16	23	27
7 Days	22	33	37
28 Days	33	43	53

The addition of pozzolana to cement causes less heat of hydration.
Pozzolana is essentially a silicious material containing clay up to 80%.

2) Rapid Hardening Cement:-IS:8041-1990

It harden fast but setting time are same as OPC.
Three days strength is equivalent to 7 days strength of OPC.
Rapid hardening property is achieved by
i) Higher C_3S content.
ii)By finer grinding.
iii) by adding calcium chloride in cement.
It has higher resistance to freezing and thawing effect comapared to OPC

Uses:

i) Used where high early strength is desired.
ii) For constructing road pavement.
iii) For cold weather concreting.
iv) Used where formwork is to be removed as early as possible.
v) It should not be used for massive concrete structure.

GEOTECHNICAL ENGINEERING

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1 PROPERTIES OF SOIL

Origin of Soil

Soil Name	Agency	Available Locations
Alluvial Soil	Running water	On the banks of river
Lacustrine Soil	lake water	On beds of lakes and ponds
Marine Soil	Sea Water	On Sea shores
Glacial Soil (Till)	Glaciers	Glaciers
Aeolian Soil	Wind	Deserts

Common Types of Soils

Loess: These are wind-blown uniformly graded fine soil. Loess is formed in arid and semi-arid regions. Its colour is yellowish brown and deposits of this soil are found in Rajasthan and North Gujarat.

Caliche: It is cemented soil rich in calcium carbonate consisting of gravel, sand and clays. These are also wind-blown in semi-arid climate and later on cemented by the calcium carbonate left out from the evaporation of capillary water.

Loam: It is a mixture of sand, silt and clay in definite proportion which in some cases may consist of organic matter.

Cumulose: Peaty (organic) soils are also called cumulose soil or muck. These are formed due to accumulation of organic content under waterlogged condition. It is generally found in the areas having deficient sewerage facilities or found after over flooding of the rivers.

Humus: Humus is a mixture of mud and dead plants. The tiny pieces of rock and humus joint to make various soils.

Peat: It is highly organic soil containing almost decomposed vegetable matter.

Tuff: These are small grained slightly cemented volcanic ash that has been transported by wind or water.

Bentonite: It is a clay formed by chemical weathering of volcanic ash which have high content of montmorillonite. Pulverized slurry of bentonite is highly plastic and is often used as a lubricant in drilling.

Kaolin (China Clay): It is very pure form of white clay, which is extensively used in ceramic industry.

Hardpans: Hardpans are types of soils that offer great resistance to the penetration of drilling tools during soil exploration. These are generally dense, well graded, cohesive aggregates of mineral particles.

Varved Clays: These are sedimentary deposits consisting of alternate thin layer of silt and clay. These clays are the result of deposition in lakes during periods of alternate high and low waters.

Till: It is formed by glaciers and iceberg and may contain mixture of gravel, sand, silt and clay. These soils are well graded.

Basic Definitions

Water Content (w):(moisture content)

- Water content (w) is also called moisture content.
- It is the ratio of weight of water to the weight of soil solids.

$$W = \frac{W_w}{W_s}; w \geq 0$$

Degree of Saturation(S)

- It is the ratio of the volume of water to the volume of voids in the soil mass.

$$S = \frac{V_w}{V_v} \times 100$$

Where, V_w = volume of water
V_v = volume of voids

- For dry soil, $S = 0\%$ and for fully saturated soil $S = 100\%$, whereas partially saturated soil have $0 < S < 100\%$.

Void Ratio (e)

- It is the ratio of the total volume of voids to the volume of solids.

$$e = \frac{V_v}{V_s}; \qquad e > 0$$

- In general, $e > 0$,.
- Void ratio of fine grained soils are generally higher than those of coarse grained soils.

Porosity (n)

- It is the ratio of volume of voids to the total volume of soil.

$$n = \frac{V_v}{V} \times 100\%$$

- The range of porosity is $0 < n < 100\%$.

Air Content (a_c)

- It is the ratio of the volume of air to the total volume of voids present in soil.

$$a_c = \frac{V_a}{V_v}$$

Percentage Air Voids (n_a)

- It is defined as the ratio of volume of air to the total volume of soil mass.

$$n_a = \frac{V_a}{V} \times 100$$

Unit Weights

a) Bulk Unit Weight (γ_t):

$$\gamma_t = \frac{W}{V}$$

b) Dry Unit Weight (γ_d):

$$\gamma_d = \frac{W_{dry}}{V}$$

c) Saturated Unit, Weight (γ_{sat}):

$$\gamma_{sat} = \frac{W_{sat}}{V}$$

d) Submerged Unit Weight or Buoyant Unit Weight (γ'_{sub} or γ'):

$$\gamma' = \frac{W_{sub}}{V} = \gamma_{sat} - \gamma_w$$

- γ' is roughly $\frac{1}{2}$ of saturated unit weight (γ_{sat}).

e) Unit Weight of Water (γ_w):

$$\gamma_w = \frac{W_w}{V_w}$$

f) Unit Weight of Solids (γ_s):

$$\gamma_s = \frac{W_s}{V_s}$$

Specific Gravity (G)

1. True or Absolute Specific Gravity:

$$G = \frac{W_s}{W_w} = \frac{\gamma_s}{\gamma_w}$$

2. Apparent or Mass Specific gravity:

$$(G_m) = \frac{\gamma_b}{\gamma_w}$$

Range of "G":

For Inorganic Soils	2.6~2.9
For Organic Soils	1~2

Some Important Relationships

1. $W_S = \frac{W}{1+W}$
2. $Se = wG$
3. $\gamma_{sat} = \left(\frac{G_s+e}{1+e}\right)\gamma_w$
4. $\gamma' = \left(\frac{G-1}{1+e}\right)\gamma_w$
5. $\gamma_d = \frac{\gamma_t}{1+w}$
6. $n = \frac{e}{1+e}$
7. $\gamma_t = \left(\frac{G_s+Se}{1+e}\right)\gamma_w$
8. $\gamma_d = \frac{G\gamma_w}{1+e}$
9. $\gamma_d = \frac{(1-n_a)G\gamma_w}{1+wG}$

Relative Density $(I_D \text{ or } D_r)$

- Relative density is also called degree of density or density index.
- It is the index which quantifies the degree of packing between the loosest and densest packing or coarse grained soil.
- The range of density index, I_D is $0 \le I_D \le 1$.

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}}$$

where, e_{max} = void ratio in loosest state

e_{min} = Void ratio in densest state

e = Void ratio in natural state

- It is the most important property of a coarse grained soil.

APPLIED MATHEMATICS

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FORMULAE

Trigonometric formulae

Trigonometric identities :

1. $sin^2\theta + cos^2\theta = 1$
2. $1 + tan^2\theta = sec^2\theta$
3. $1 + cot^2\theta = cosec^2\theta$

Compound angel formulae:

1. $sin(A \pm B) = sin\,A\,cos\,B \pm cos\,A\,sin\,B$
2. $cos(A \pm B) = cos\,A\,cos\,B \mp sin\,A\,sin\,B$
3. $tan(A \pm B) = \frac{tan\,A \pm tan\,B}{1 \pm tan\,A\,tan\,B}$

Factorization formulae:

1. $sin\,C + sin\,D = 2sin\left(\frac{C+D}{2}\right)cos\left(\frac{C-D}{2}\right)$
2. $sin\,C - sin\,D = 2cos\left(\frac{C+D}{2}\right)sin\left(\frac{C-D}{2}\right)$
3. $cos\,C + cos\,D = 2cos\left(\frac{C+D}{2}\right)cos\left(\frac{C-D}{2}\right)$
4. $cos\,C - cos\,D = -2sin\left(\frac{C+D}{2}\right)sin\left(\frac{C-D}{2}\right)$

De-factorization formulae

1. $2\,sin\,A\,cos\,B = sin(A+B) + sin(A-B)$
2. $2\,cos\,A\,sin\,B = sin(A+B) - sin(A-B)$
3. $2\,cos\,A\,cos\,B = cos(A+B) + cos(A-B)$
4. $2\,sin\,A\,sin\,B = cos(A-B) - cos(A+B)$

Double angle formulae:

1. $sin\,2A = 2\,sin\,A\,cos\,A = \frac{2\,tan\,A}{1+tan^2A}$
2. $cos\,2A = cos^2A - sin^2A = \frac{1-tan^2A}{1+tan^2A}$
3. $tan\,2A = \frac{2\,tan\,A}{1-tan^2A}$

Triple angle formulae:

1. $sin\,3A = 3\,sin\,A - 4\,sin^3A$
2. $cos\,3A = 4\,cos^3A - 3\,cos\,A$
3. $tan\,3A = \frac{3\,tan\,A - tan^3A}{1-3\,tan^2A}$

Inverse Trigonometric identities:

1. $sin^{-1}A + cos^{-1}A = \frac{\pi}{2}$
2. $cosec^{-1}A + \sec^{-1}A = \frac{\pi}{2}$

Commonly used formulae

1. $sin^2A = \frac{1-cos\,2A}{2}$
2. $cos^2A = \frac{1+cos\,2A}{2}$

Derivative formulae:

1. $\frac{d}{dx}X^n = nX^{n-1}$
2. $\frac{d}{dx}1/X = -1/X^2$
3. $\frac{d}{dx}e^X = e^X$
4. $\frac{d}{dx}a^X = a^X log a$
5. $\frac{d}{dx}\,log\,X = 1/X$
6. $\frac{d}{dx}\,log_a X = \frac{1}{X \times \log a}$
7. $\frac{d}{dx}sin\,X = cos\,X$
8. $\frac{d}{dx}cos\,X = -sin\,X$
9. $\frac{d}{dx}tan\,X = sec^2X$
10. $\frac{d}{dx}cot\,X = -cosec^2X$
11. $\frac{d}{dx}\,sec\,X = sec\,X\,tan\,X$
12. $\frac{d}{dx}cosec\,X = -cosec\,X\,cot\,X$
13. $\frac{d}{dx}\,sin^{-1}X = \frac{1}{\sqrt{1-x^2}}$
14. $\frac{d}{dx}\,tan^{-1}X = \frac{1}{1+x^2}$
15. $\frac{d}{dx}\,sec^{-1}X = \frac{1}{|x|\sqrt{x^2-1}}$
16. $\frac{d}{dx}(u \times V) = u\frac{dV}{dX} + u\frac{du}{dX}$
17. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dX} - u\frac{dV}{dX}}{V^2}$

Integration formulae

1. $\int sinX\;dx = -cosX + C$
2. $\int cosX\;dx = sinX + C$
3. $\int tanX\;dx = log|secX| + C$
4. $\int cotX\,dx = log|sinX| + C$
5. $\int secX\;dx = log|secX + tanX| + C$
6. $\int cosecX\;dx = log|cosecX - cotX| + C$
7. $\int \sec^2X\,dx = tanX + C$
8. $\int \mathrm{cosec}^2X\;dx = -cotX + C$
9. $\int secX\,tanX\,dx = secX + C$
10. $\int cosecX\,cotX\;dx = -cosecX + C$
11. $\int \log x\;dx = x\log x - x$
12. $\int X^n\;dx = \frac{x^{n+1}}{n+1} + C$
13. $\int 1/X\;dx = logX + C$
14. $\int e^X\;dx = e^X + C$
15. $\int a^X\;dx = \frac{a^X}{log a} + C$
16. $\int \frac{1}{\sqrt{a^2-x^2}}\;dx = \sin^{-1}\frac{x}{a} + C$
17. $\int \frac{1}{a^2+x^2}\;dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$
18. $\int \frac{1}{\sqrt{x^2-a^2}}\;dx = logx + \sqrt{x^2-a^2} + C$
19. $\int \frac{1}{\sqrt{x^2+a^2}}\;dx = logx + \sqrt{x^2+a^2} + C$
20. $\int \frac{1}{a^2-x^2}\;dx = \frac{1}{2a}log\left|\frac{a+x}{a-x}\right| + C$
21. $\int \frac{1}{x^2-a^2}\;dx = \frac{1}{2a}log\left|\frac{x-a}{x+a}\right| + C$

22. $\int \frac{1}{\sqrt{a^2-x^2}}\ dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$
23. $\int \sqrt{x^2-a^2}\ dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left(x+\sqrt{x^2-a^2}\right) + C$
24. $\int \sqrt{x^2+a^2}\ dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left(x+\sqrt{x^2+a^2}\right) + C$
25. $\int e^x[f(X)+f'(X)]\,dX = e^X f(X)$
26. $\int \frac{f'(X)}{f(x)}dX = logf(X)$
27. $\int \frac{f'(X)}{\sqrt{f(x)}}dX = 2\sqrt{f(x)}$
28. $\int [f(x)]^n f'^{(X)}dX = \frac{f(x)^{n+1}}{n+1}$
29. $\int e^{f(x)} f'^{(X)}dX = e^{f(x)}$
30. $\int e^{aX} cosbX\ dX = \frac{e^{aX}}{a^2+b^2}(acosbX + bsinbX)$
31. $\int e^{aX} sinbX\ dX = \frac{e^{aX}}{a^2+b^2}(asinbX - bcosbX)$
32. $\int u\,v\,dX = u\int v\,dX - \int\left[\frac{du}{dX}\int v\,dX\right]dX$
33 $\int u\,v\,dX = u\,v_1 - u\,v_2 + u"v_3 - \ldots$

Reduction Formulae

1. $\int_0^{\frac{\pi}{2}} sin^n x dx = \frac{n-1}{n}\times\frac{n-3}{n-2}\times\frac{n-5}{n-4}\times----\times p$
$n\ is\ even \rightarrow p = \frac{\pi}{2}$
$n\ is\ odd \rightarrow p = 1$
2. $\int_0^{\frac{\pi}{2}} cos^n x dx = \frac{n-1}{n}\times\frac{n-3}{n-2}\times\frac{n-5}{n-4}\times----\times p$
$n\ is\ even \rightarrow p = \frac{\pi}{2}$
$n\ is\ odd \rightarrow p = 1$
3. $\int_0^{\pi} sin^n x dx = 2\int_0^{\frac{\pi}{2}} sin^n x dx$ for all n
4. $\int_0^{\pi} cos^n x dx$
$= 2\int_0^{\frac{\pi}{2}} cos^n x dx \quad n\ is\ even\ number = 0$
$n\ is\ odd\ number$
5. $\int_0^{2\pi} sin^n x dx = 4\int_0^{\frac{\pi}{2}} sin^n x dx \rightarrow n\ is\ even = 0$
$n\ is\ odd\ number$
6. $\int_0^{2\pi} cos^n x dx = 4\int_0^{\frac{\pi}{2}} sin^n x dx \rightarrow n\ is\ even = 0$
$n\ is\ odd\ number$
7. $\int_0^{\pi} sin^n cosx dx = \int_0^{\frac{\pi}{2}} sin\ cosx dx = \frac{1}{1+n}$

Rules of Definite Integral

1. $\int_a^b f(x)dx = \int_a^b f(t)dt$
2. $\int_a^b f(x)dx = -\int_a^b f(t)dx$
3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx; a < c < b$
4. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
5. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
6. $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx \ \ when\ f(x)\ is\ even$
$= 0\ \ when\ f(x) is\ odd$

Length of curves

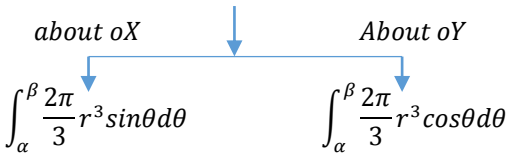
1. $y = f(x)\ between\ x = a, x = b$
$\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$
2. $x = g(y) between\ g = a, y = b$
$\int_a^b \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$
3. $r = f(\theta)\ between\ \theta = \alpha\ to\ \theta = \beta$
$\int_\alpha^\beta \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$
4. $x = f(t), y = g(t) between\ t = t_1 \& t = t_2$
$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)+\left(\frac{dy}{dt}\right)^2}\,dt$

Areas of Curves

1. $Area\ bounded\ by\ y = f(x), x-axis\ and\ x = a, x = b\ is\ \int_a^b ydx$
2. $Area\ bounded\ by\ x = g(y), y-axis\ y = a, y = b\ is\ \int_a^b xdx$
3. $Area\ bounded\ by\ r = f(\theta) between\ \theta = \alpha, \theta = \beta\ is\ = \frac{1}{2}\int_\alpha^\beta r^2 d\theta$

Volume of revolution

1. Volume of revolution about x-axis of area bounded by y=f(x), x axis & x=a, x=b is
$$v = \int_a^b \pi y^2 dx$$
2. Volume of revolution about y-axis of area bounded by x=f(y), g axis, y=a, y=b is
$$v = \int_a^b \pi x^2 dy$$
3. Volume of revolution bounded by
$$r = f(\theta)\ and\ \theta = \alpha, \theta = \beta$$



1. MATRIX

The theory of matrices is one of the powerful tools in applied mathematics as well as in various branches of engineering. The study of matrices is motivated by the well-known problem of solving the system of linear equations which is useful to solve many problems of engineering.

"Matrix is an array of m x n entries in m rows and n columns enclosed by square bracket".

It is denoted by single capital letter A, B and C etc. and entries of matrix are denoted by small letters or numbers. Matrix with m rows and n columns is called as Matrix of order $m \times n$ (Read as m by n) and denote it as $A_{m\times n}$

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots . a_{1n} \\ a_{21} & a_{22} & \ldots a_{2n} \\ a_{31} & a_{32} & \ldots . a_{3n} \\ \vdots & \cdot & \cdot \\ & \cdot & \cdot \\ \vdots & \cdot & \cdot \\ a_{m1} & a_{m2} & a_{mn} \end{bmatrix}$$

The matrix $A_{m\times n}$ can be denoted as $[a_{ij}]$, where, i =1,2,3,...,m and j =1,2,3,...,n

1.1 Elementary Operations

1. Equality of matrices:

Two matrices A and B are said to equal if both are of same order and their corresponding elements are equal, i.e., if A = $[a_{ij}]_{m\times n}$ and B = $[b_{ij}]_{m\times n}$

Then $A = B \; if \quad a_{ij} = b_{ij} \; \forall_{i,j}$

2. Addition and Subtraction of matrices:

If A = $[a_{ij}]_{m\times n}$ and B = $[b_{ij}]_{m\times n}$ are two matrices of the same order then their sum is denoted by A + B and defined as A + B = $[a_{ij} + b_{ij}]_{m\times n}$ and is obtain by adding the corresponding elements of A and B.

3. Multiplication of matrix by Scalar:

Let A = $[a_{ij}]_{m\times n}$ be any matrix and k be a non zero scalar then kA = B = $[b_{ij}]_{m\times n}$, i.e., multiplication of matrix A by scalar k is nothing but multiplying each element of matrix A by scalar k.

E.g. If A = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Then 4A = $\begin{bmatrix} 4(1) & 4(-1) \\ 4(2) & 4(3) \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix}$

Note that:

i) we defined subtraction of two matrices as $A - B = A + (-B)$

ii) Order of matrices A and kA, $A + B \; and \; A - B$ are always same. For any matric es A, B and C, we have following properties;

1. $A + B = B + A$
2. $A + (B + C) = (A + B) + C$
3. $(k_1.k_2)A = k_1(k_2A) = k_2(k_1A)$
4. $k_1(A + B) = k_1A + k_1B$
5. $Z + A = A + Z = A$, for zero matrix Z.

Where k_1, k_2 are constants.

4. Trace of matrix :

Let A be any square matrix, then the trace of matrix A is denoted by $tr(A)$ and defined as the sum of elements on principal diagonal of matrix A.

e.g. : If A = $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$,then $tr(A) = 1 + 4 = 5$

5. Multiplication of matrices :

Let $A = [a_{ij}]_{m\times n} and \; B = [b_{ij}]_{n\times p}$ be any two matrices. then their product AB is a matrix

$C = [c_{ij}]_{m\times n}$, where the entry c_{ij} is sum of the products of elements in ith row of matrix A with the corresponding elements in jth column of matrix B.

Note that:

1. Matrix multiplication is defined only if number of column in first matrix is equal to number of rows in second matrix.
2. Matrix multiplication is not commutative, i.e., $AB \neq BA$
3. Matrix multiplication is associative, i.e., $A(BC) = (AB)C$
4. $k(AB) = (kA)B = A(kB)$
5. $AA = A^2, AAA = (AA)A = A^3$ and so on...
6. If I is identity matrix, then $AI = IA = A \; and \; I = I^2 = I^3 = \ldots = I^n$

6. Transpose of Matrix

The new matrix obtained by interchanging the rows and columns of the original matrix is called as the transpose of the matrix. If A = $[a_{ij}]$ be an m × n matrix, then the matrix obtained by interchanging the rows and columns of A would be the transpose of A. of It is denoted by A'or (A^T). In other words, if A = $[a_{ij}]_{mxn}$,thenA' = $[a_{ji}]_{nxm}$. For example,

$$A = \begin{bmatrix} 3 & -5 \\ 4 & 7/2 \\ 8 & 5/8 \end{bmatrix} \qquad A' = \begin{bmatrix} 3 & 4 & 9 \\ -5 & 7/2 & 5/8 \end{bmatrix}$$

7. Determinant

- If A is any square matrix then determinant of matrix A is denoted by |A|,

– For 2×2 matrix A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2}$

Then $|A| = (a_{11} \times a_{22}) - (a_{12} \times a_{21})$

- For 3×3 matrix A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$

Then $|A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{23} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

$$|A| = a_{11}(a_{22}.a_{33} - a_{23}.a_{32}) - a_{12}(a_{21}.a_{33} - a_{23}.a_{31}) + a_{13}(a_{21}.a_{32} - a_{22}.a_{31})$$

Note that, the $\det(A) = \det(A^T)$

Ex. 1. Find determinant of A = $|A| = \begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix}$

We have determinant of A = $|A| = \begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix}$

$|A| = 12 - 10 \Rightarrow |A| = 2$

8. Inverse of matrix:

Let A be any square matrix of order n, if there exists a matrix B such that $AB = BA = I$, then B is said to be the inverse of matrix A.

The inverse of matrix A is denoted by A^1, therefore $AA^1 = A - 1A = I$. If inverse of matrix A exists then A is called invertible.

- A^{-1} is always unique.

–If A is invertible, then $A^{-1}$is also invertible and $(A^{-1})^{-1}$ = A

–Reversal law : $(AB)^{-1} = B^{-1}A^{-1}$

– $(A^n)^{-1} = (A^{-1})n = A^{-n}$, for any n

–Cancellation law : AB = AC then B = C.

Types of Matrices

1. Row Matrix:

A matrix having only one row is called as "Row Matrix or Row Vector".

e.g. $[1 \quad 2 \quad 1], [1 \quad 0 \quad 2 \quad 3 \quad -3], [a]$ etc.

2. Column Matrix :

A matrix having only one column is called as "Column Matrix or Column vector".

e.g. : $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}, [b]$ etc.

Note that : A row or column matrix is also called as vector.

3. Null or Zero Matrix:

A matrix containing only zeroes is called as "Null or Zero Matrix".

e.g. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, [0]$ etc.

4. Square Matrix:

A matrix is said to be "Square Matrix", if number of rows is equal to number of column in it.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}, \begin{bmatrix} 3 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{bmatrix}_{3\times3}, [4]_{1\times1}$$ etc.

5. Diagonal Matrix:

A square matrix is said to be "Diagonal Matrix", if all its non-diagonal entries are zero and atleast one diagonal element are non zero.

e.g. $\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}, etc$

6. Scalar Matrix :

A diagonal matrix in which all entries on principal diagonal are equal is called as "Scalar Matrix".

e.g. : e.g. $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, etc$

7. Unit Matrix or Identity Matrix:

A scalar matrix in which all entries on principal diagonal are unit (one) is called as "Unit or Identity Matrix".

i.e., $[a_{ij}]$ is a Identity, if $a_{ij} = \begin{cases} 1 & if\ i = j \\ 0 & if\ i \neq j \end{cases}$

e.g. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ etc

8. Upper Triangular Matrix:

A matrix in which all entries below principal diagonal are zero is called as "Upper triangular matrix".

i.e., $[a_{ij}]$ is a upper triangular, iff $a_{ij} = 0\ if\ i > j$

9. Lower Triangular Matrix:

A matrix in which all entries above principal diagonal are zero is called as "Lower triangular matrix".

i.e., $[a_{ij}]$ is a lower triangular, iff $a_{ij} = 0\ if\ i < j$